Optical Flow

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1 Introduction

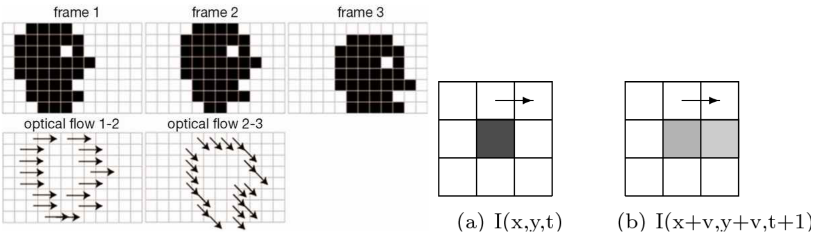

Figure 1: Simple illustrations of optical flows

Optical flow (or optic flow) is the pattern of apparent motion of objects, surfaces, and edges in a visual scene caused by the relative motion between an observer and a scene. This concept was first introducted by Gibson in 1950. Its name was from the fact that the observation of moving objects forms a series of continuously changing images on the retina of the our eyes and this series of continuously changing information ”flow” through the retina (the image plane) as if it were a light ”flow”.

To summarize, the purpose of studying the optical flow is to estimate the motion field(which is the movement of objects in the three-dimensional real world) that can not be read directly from image sequences. And the optical flow is the projection of the motion field on the two-dimensional image plane.

2 Modeling

We intend to calculate the motion between two image frames $I(t)$ and $I(t+dt)$ which are taken at times t and $t+dt$ at every pixel position $\vec{u} = (u, v) = (\frac{dx}{dt}, \frac{dy}{dt})$. We have the following hypothesis:

- Brightness constancy constraint: we don't need to consider the RGB value's change between two images
- Small motion: we can find the corresponding point of a pixel within a very little neighborhood

Owing to brightness constancy constraint:

$$I(x, y, t) = I(x + dx, y + dy, t + dt)$$

Owing to small motion, we use the first order Taylor expansion:

$$I(x, y, t) = I(x, y, t) + \frac{\partial I}{\partial x}dx + \frac{\partial I}{\partial y}dy + \frac{\partial I}{\partial t}dt$$

And then we could get the Constraint Equation of Optical flow:

$$I_X dx + I_y dy + I_t dt = 0 \tag{1}$$
$$I_X u + I_y v + I_t = 0 \tag{2}$$

In the following, we will use different methods to solve this problem.

3 Lucas–Kanade method

Hypothese The displacement of the image contents between two nearby frames is approximately constant within a neighborhood of the point p under consideration.

Algorithm For a block with a pixel size of $k \times k$ (typically between 2 and 7), we can get k^2 corresponding equations:

$$\begin{cases} I_x(p_1)u + I_y(p_1)v & = -I_t(p_1) \\ I_x(p_2)u + I_y(p_2)v & = -I_t(p_2) \\ & \dots \\ I_x(p_{k^2})u + I_y(p_{k^2})v & = -I_t(p_{k^2}) \end{cases} \rightarrow \begin{bmatrix} I_x(p_1) & I_y(p_1) \\ I_x(p_2) & I_y(p_2) \\ \dots & \dots \\ I_x(p_{k^2}) & I_y(p_{k^2}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -I_t(p_1) \\ -I_t(p_2) \\ \dots \\ -I_t(p_{k^2}) \end{bmatrix} \rightarrow Ax = b$$

We use the least square method to solve the equation: $\hat{x} = argmin\|Ax - b\|^2$ and the optimal solution is $\hat{x} = (A^T A)^{-1} A^T b$. That is:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum_i I_x(p_i)^2 & \sum_i I_x(p_i)I_y(p_i) \\ \sum_i I_y(p_i)I_x(p_i) & \sum_i I_y(p_i)^2 \end{bmatrix} \begin{bmatrix} -\sum_i I_x(p_i)I_t(p_i) \\ -\sum_i I_y(p_i)I_t(p_i) \end{bmatrix}$$

Advantage

1. Easy and fast calculation
2. Accurate time derivatives

Disadvantage

1. Errors on boundaries of moving object
2. The hypothesis is strong

4 Horn-Schunck Method

We consider that he optical flow of each pixel can be different, and the optical flow (u, v) can be changed during the motion of an object in the video.

Global Energy Function The global energy function is composed by two parts:

- Point-wise energy: $E_d(i,j) = (I_x u_{ij} + I_y v_{ij} + I_t)^2$
- Pair-wise energy: $E_s(i,j) = \frac{1}{4}[(u_{ij} - u_{i+1,j})^2 + (u_{ij} - u_{i,j+1})^2 + (v_{ij} - v_{i+1,j})^2 + (v_{ij} - v_{i,j+1})^2]$

So we conclude the objective function is

$$\min_{u,v} \sum_{i,j} (E_d(i,j) + \lambda E_s(i,j))$$

Gradient The gradient in matrix form of kth iteration is:

$$u^{k+1} = \bar{u}^k - \frac{I_x(I_x\bar{u}^k + I_y\bar{v}^k + I_t)}{\lambda + I_x^2 + I_y^2}$$
$$v^{k+1} = \bar{v}^k - \frac{I_x(I_x\bar{u}^k + I_y\bar{v}^k + I_t)}{\lambda + I_x^2 + I_y^2}$$

where $\bar{u}^k(x,y) = \frac{1}{4}(u_{i-1,j} + u_{i+1,j} + u_{i,j+1}, u_{i,j-1})$, $\bar{v}^k(x,y) = \frac{1}{4}(v_{i-1,j} + v_{i+1,j} + v_{i,j+1}, v_{i,j-1})$

Advantage

1. Smooth flow
2. Global information
3. Accurate time derivatives and more flexibility

Disadvantage

1. Iterative method: slow
2. Unsharp boundaries

5 Applications

Optical flow is crucial in artificial vision because it compute the relative speed of objects. The field of application is wide. Drones and Automatics cars use it to detect structures and the path of pedestrian and other cars. It can also detect speed anomalies in a process. In addition it can be used for compressing video purposes by summing up the movement of objects.

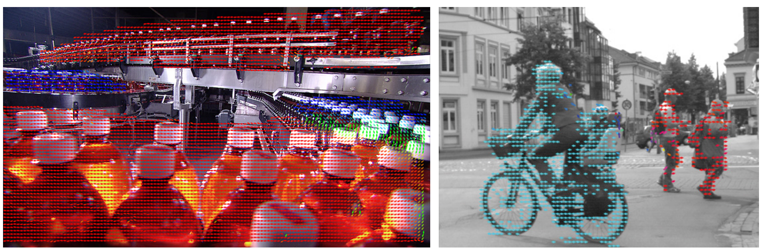

Figure 2: Several applications of the optical flow

References

[1] Wikipedia, Optical flow
https://en.wikipedia.org/wiki/Optical_flow

[2] Eric Yuan's Blog, Coarse-to-fine Optical Flow (Lucas & Kanade)
http://eric-yuan.me/coarse-to-fine-optical-flow/