Variational Shape Reconstruction via Quadric Error Metrics

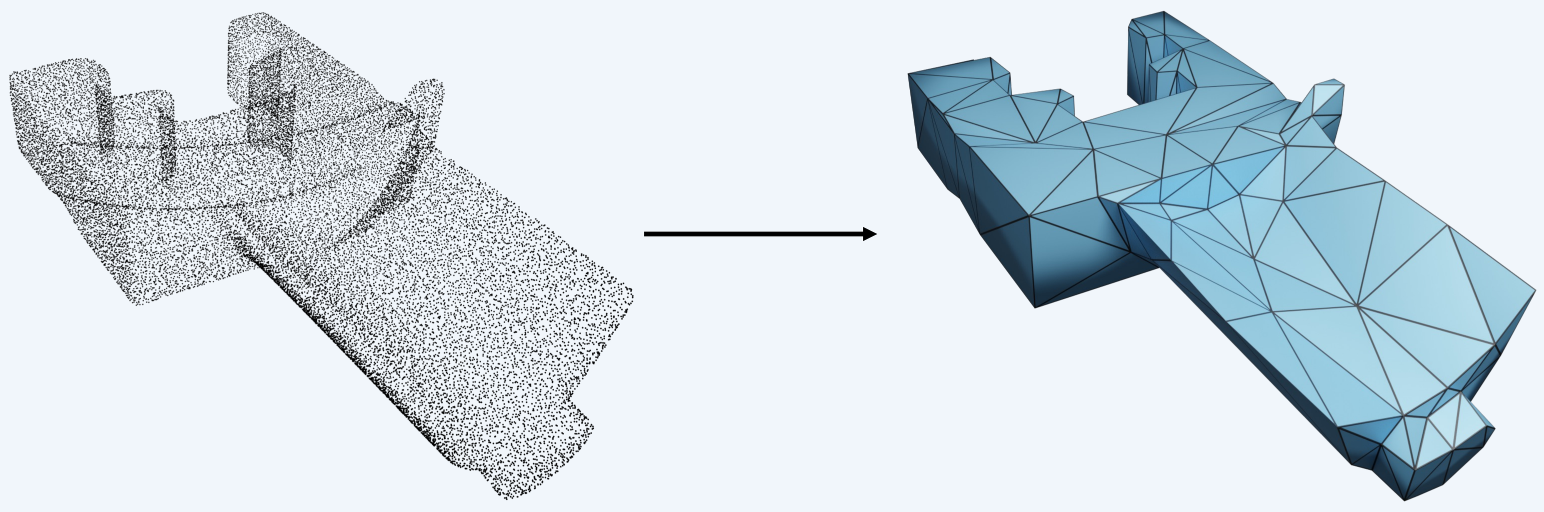
Tong Zhao, Laurent Busé, David Cohen-Steiner, Tamy Boubekeur, Jean-Marc Thiery, Pierre Alliez

WHAT WE DO ?

Input : A point cloud with/w.o. unoriented normals

Output : A **concise mesh** which is

- √ feature-perserving
- √ favoring 2-manifold
- √ orientable
- √ favoring anisotropic triangles



WHY QEM ?

Quadric error metric (QEM) is a powerful tool for mesh processing. It is:

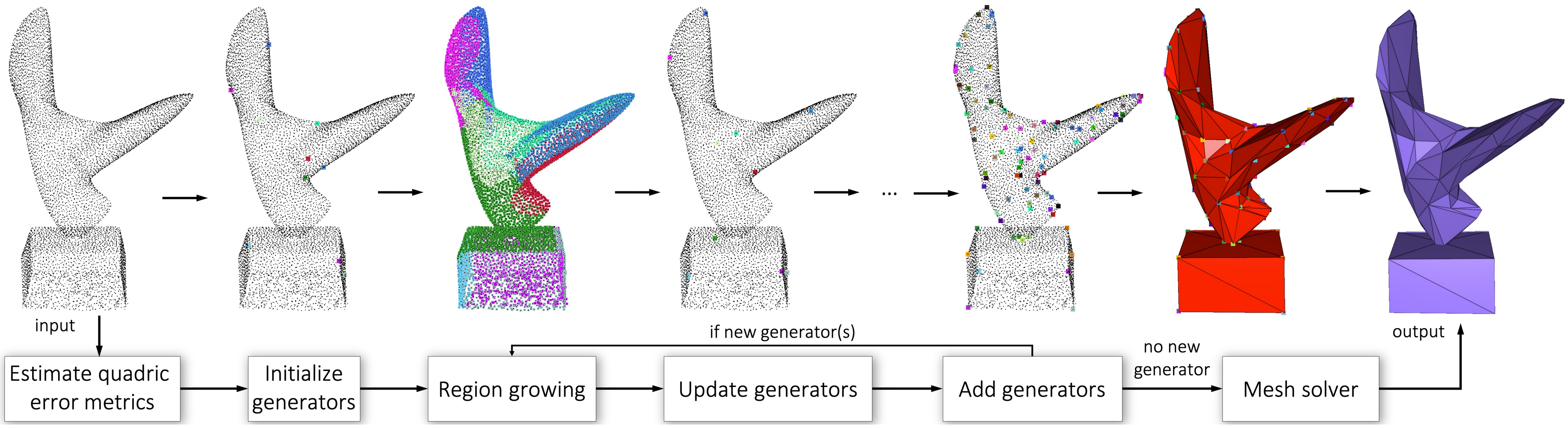
- Feature-aware
- Fast and effective
- Orientation-independant
- Robust to noise

while it is rarely used in point cloud processing.

WHY VARIATIONAL ?

Designing a **coarse-to-fine** variational method allows us to have a low memory footprint.

OUR METHOD: QEM - VSR



Step 1: Intialization

(1) Local normal and area estimation based on K-nearest neighbor graph.

Normal: Principal component analysis or Jet fitting.

Area: $a_{p_i} = \frac{1}{2k^2} \cdot \left(\sum_{p_j | (p_i, p_j) \in KNN(\mathcal{P})} \| p_i - p_j \|^2 \right)$

(2) QEM initialization

Plane quadric: $Q_{p_i} = \left(n_i^x, n_i^y, n_i^z, -n_i \cdot p_i^T\right)^T \cdot \left(n_i^x, n_i^y, n_i^z, -n_i \cdot p_i^T\right)$

Point quadric: $Q_{v_i} = \sum_{p_j | (p_i, p_j) \in KNN(\mathcal{P})} a_{p_j} \cdot Q_{p_j}$

(3) Generator initialization: random selection from input points.

Step 2: Clustering

(1) Partitioning (Expectation-Step)

The cost of adding the i^{th} point to the j^{th} cluster is:

$$E(p_i, l_j) = \underbrace{[c_j, 1]^T \cdot Q_{v_i} \cdot [c_j, 1]}_{\text{QEM error}} + \lambda \cdot \underbrace{\| p_i - c_j \|^2}_{\text{L2 error (only for ill-posed regions)}}$$

(2) Updating (Maximization-Step)

The optimal generator of the j^{th} cluster is:

$$c_j^* = \underset{p \in \mathbb{R}^3}{\operatorname{argmin}} [p, 1]^T \cdot Q_{c_j} \cdot [p, 1]$$

(3) Batch splitting

Add the point maximizing E as a new generator if criterion fails.

$$p_{max}(l_j) = \underset{p_i \in l_j}{\operatorname{argmax}} [p_i, 1]^T \cdot Q_{c_j} \cdot [p_i, 1]$$

Step 3: Meshing

(1) Edge candidate: derived from the adjacency between clusters.

(2) Face candidate selection: derived from finding 3-cycles of edge candidate set.

(3) Binary solver: edge candidates and face candidates as variables.

$$\max_{\{b_{f_1}, \ldots, b_{f_n}\}} \sum_{i=1}^{n} b_{f_i} \cdot E_{fitting}(f_i)$$

$$s.t. \quad 2b_{e_i} - \sum_{f_j \, around \, e_i} b_{f_j} = 0 \quad \text{(Manifold constraint)}$$

SIGGRAPH 2023
LOS ANGELES+ 6-10 AUG

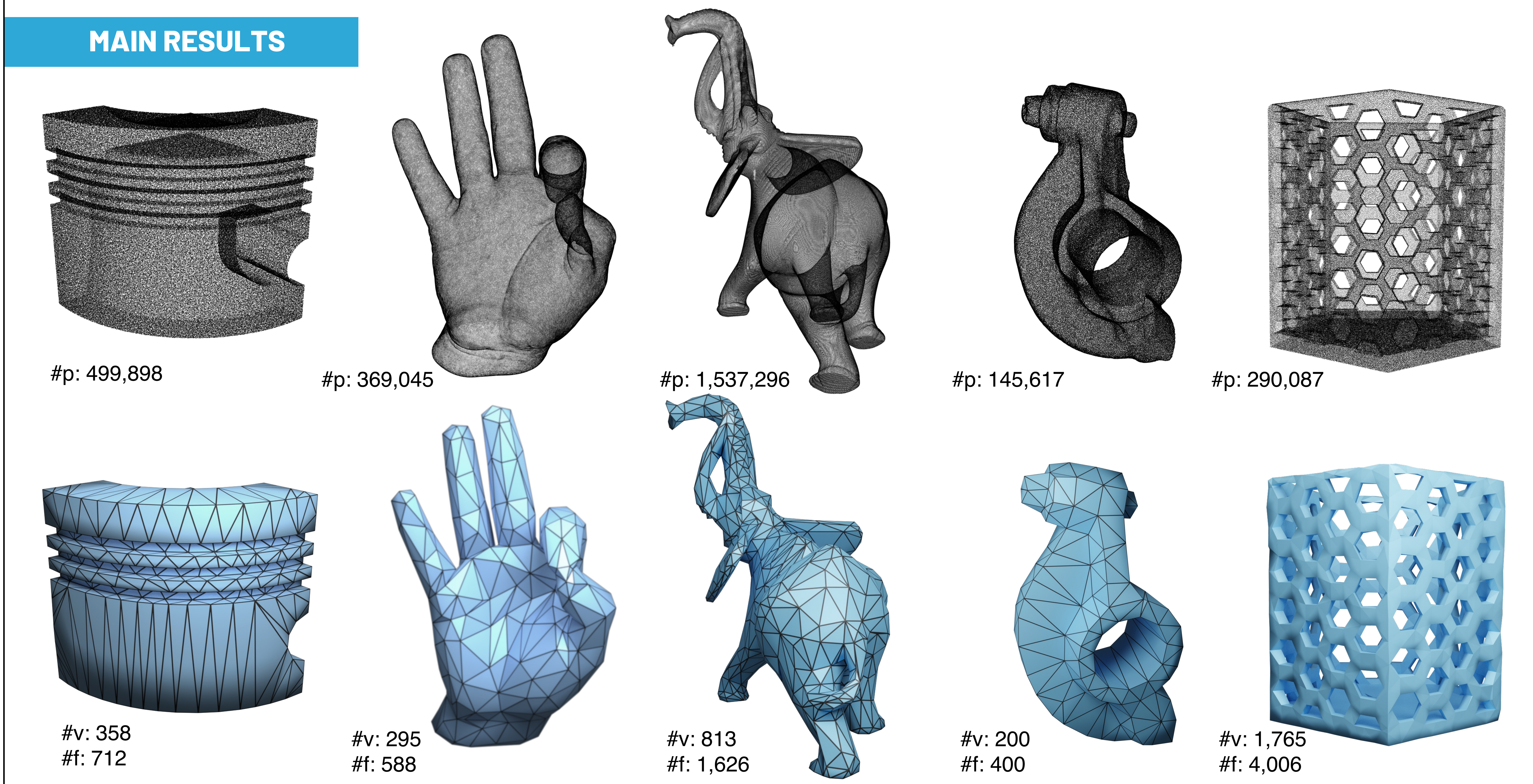
Contacts : Tong Zhao (zhaotong.patricia@gmail.com) / Pierre Alliez (pierre.alliez@inria.fr)

Variational Shape Reconstruction via Quadric Error Metrics

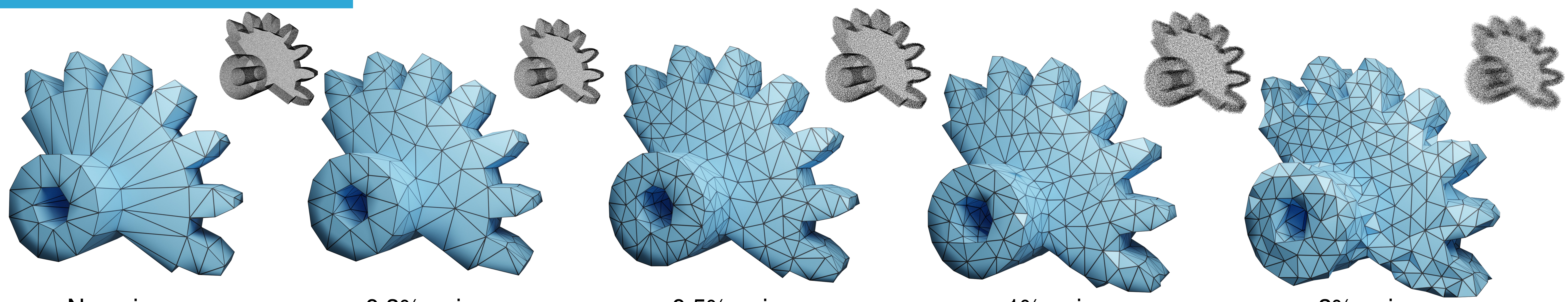
Tong Zhao, Laurent Busé, David Cohen-Steiner, Tamy Boubekeur, Jean-Marc Thiery, Pierre Alliez

3iA Côte d'Azur Interdisciplinary Institute for Artificial Intelligence · Université Côte d'Azur · Inria · Telecom Paris · Adobe

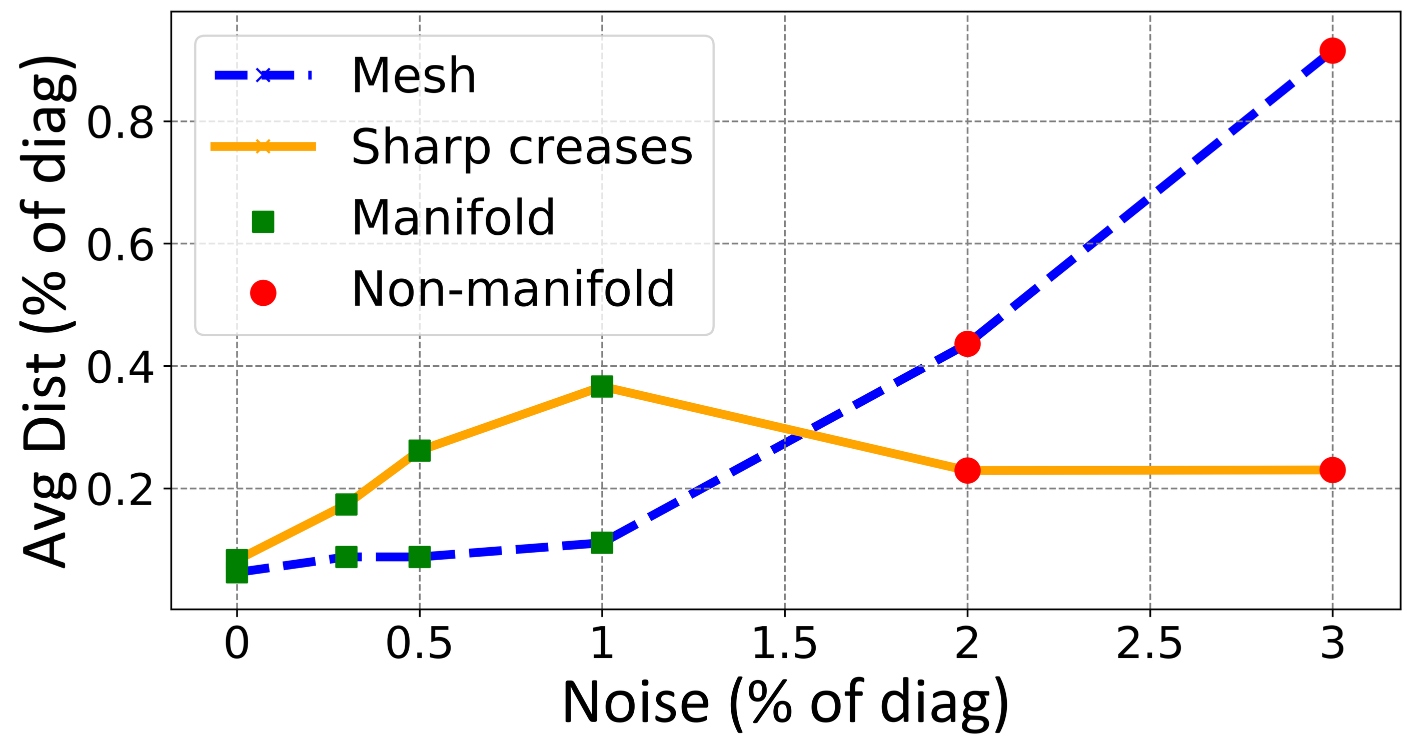
MAIN RESULTS



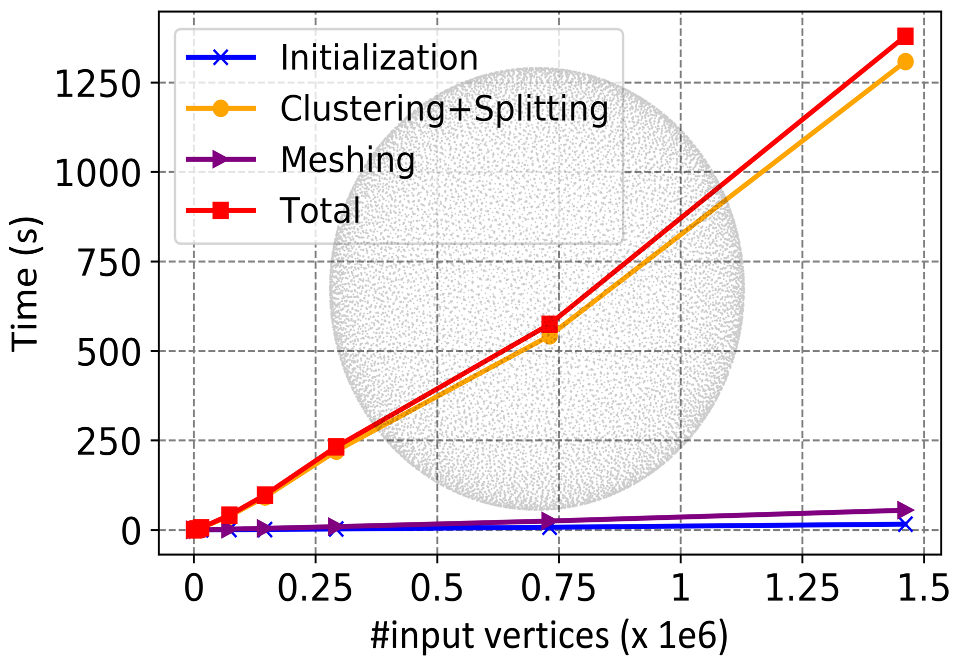
ROBUSTNESS TO NOISE

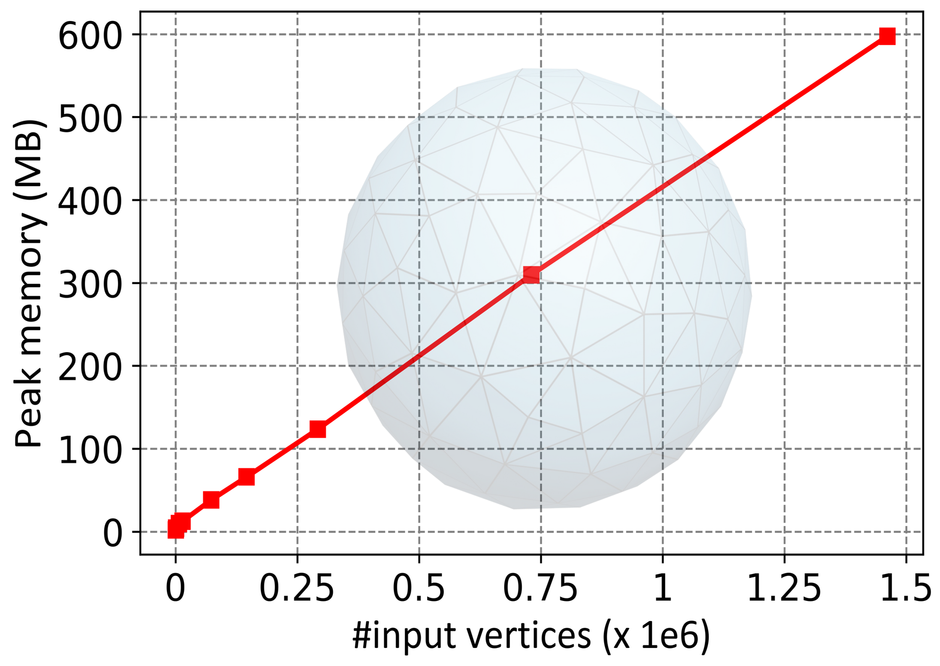


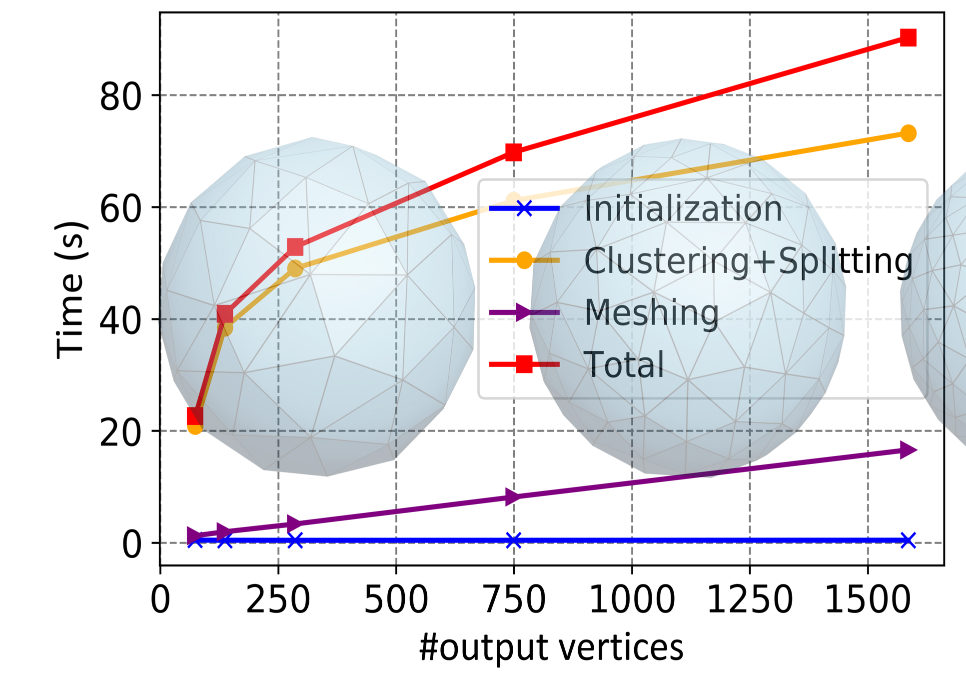
No noise | 0.3% noise | 0.5% noise | 1% noise | 2% noise

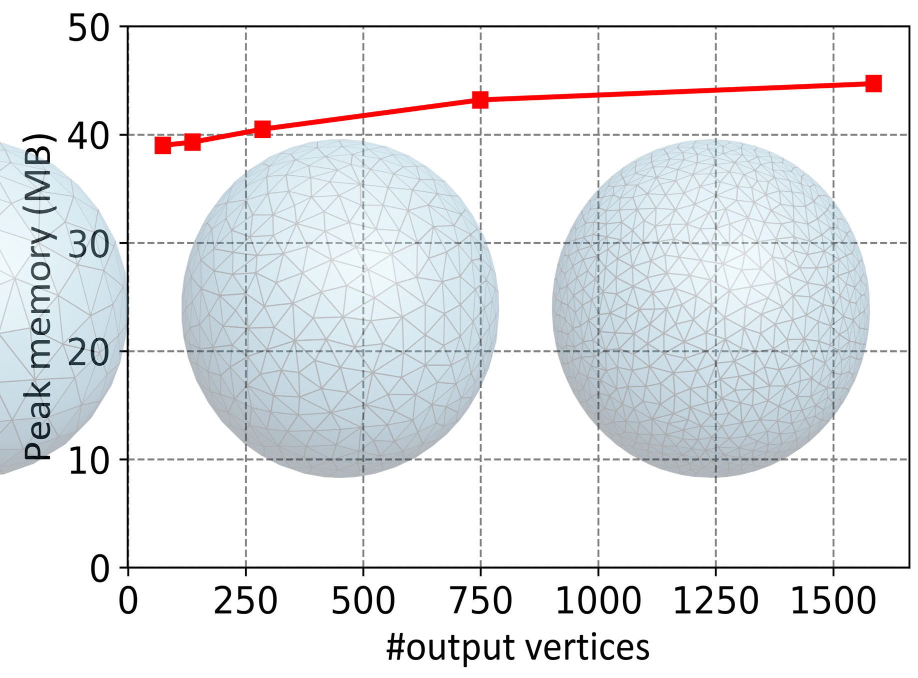


PERFORMANCE









FOLLOW US

Website: https://tong-zhao.github.io/vsr

The code will be soon released in **CGAL**!

Contacts : Tong Zhao (zhaotong.patricia@gmail.com) / Pierre Alliez (pierre.alliez@inria.fr)